Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2023

Test 1 – Differentiation Rules and Applications (10%)

3.1.7, 3.1.8, 3.1.10 – 3.1.16, 3.2.1 – 3.2.3

JO

Time Allowed	**First Name**	**Surname**	**Marks**
18 minutes			15 marks

Circle your Teacher's Name: Mrs Alvaro, Ms Chua, Mrs Fraser-Jones, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Narendranathan, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ASSUMED

QUESTION 8 **(4 marks)**

The gradient of the curve with equation $y = \frac{a}{x} + bx^2$ at the point with coordinates $(3,6)$ is 7. Calculate the values of a and b.

$6 = \frac{a}{3} + bx^2$

$6 = \frac{a}{3} + 9b$ ✓ Forms first equation

$y' = -\frac{a}{x^2} + 2bx$ ✓ Differentiates

$7 = -\frac{a}{9} + 6b$ ✓ Forms second equation

$a = -9$

$b = 1$ ✓ Solves and states values of a and b

QUESTION 9 **(3, 1, 1, 4, 2 – 11 marks)**

A man walks along a riverbank from point A to point B, a distance of x m, at a speed of 1.25 m/s. He then swims, at a speed of 1 m/s, to a point D which is directly opposite point C.

The point C is 20 m downstream from A. $DC = 10$ m. $0 \le x \le 20$

a) Show that the time taken (t seconds) to go from A to D in this way is:

$$t(x) = \sqrt{100 + (20 - x)^2} + \frac{4x}{5}$$

Time for $A \to B$:

$$1.25 = \frac{x}{t_1}$$

$$t_1 = 0.8x$$

$$= \frac{4x}{5}$$

✓ Expression for time for first leg

Dist $BD = \sqrt{(20 - x)^2 + 100}$

✓ Distance to swim

Time for $B \to D$:

$$1 = \frac{\sqrt{(20 - x)^2 + 100}}{t_2}$$

$$t_2 = \sqrt{(20 - x)^2 + 100}$$

✓ Expression for time for second leg

Therefore

$$t = \frac{4x}{5} + \sqrt{(20 - x)^2 + 100}$$

b) Find $\frac{dt}{dx}$.

$$\frac{dt}{dx} = \frac{5x + 4\sqrt{x^2 - 40x + 500} - 100}{5\sqrt{x^2 - 40x + 500}}$$

✓ Correct derivative (CAS)

OR

$$\frac{dt}{dx} = \frac{x - 20}{\sqrt{x^2 - 40x + 500}} + \frac{4}{5}$$

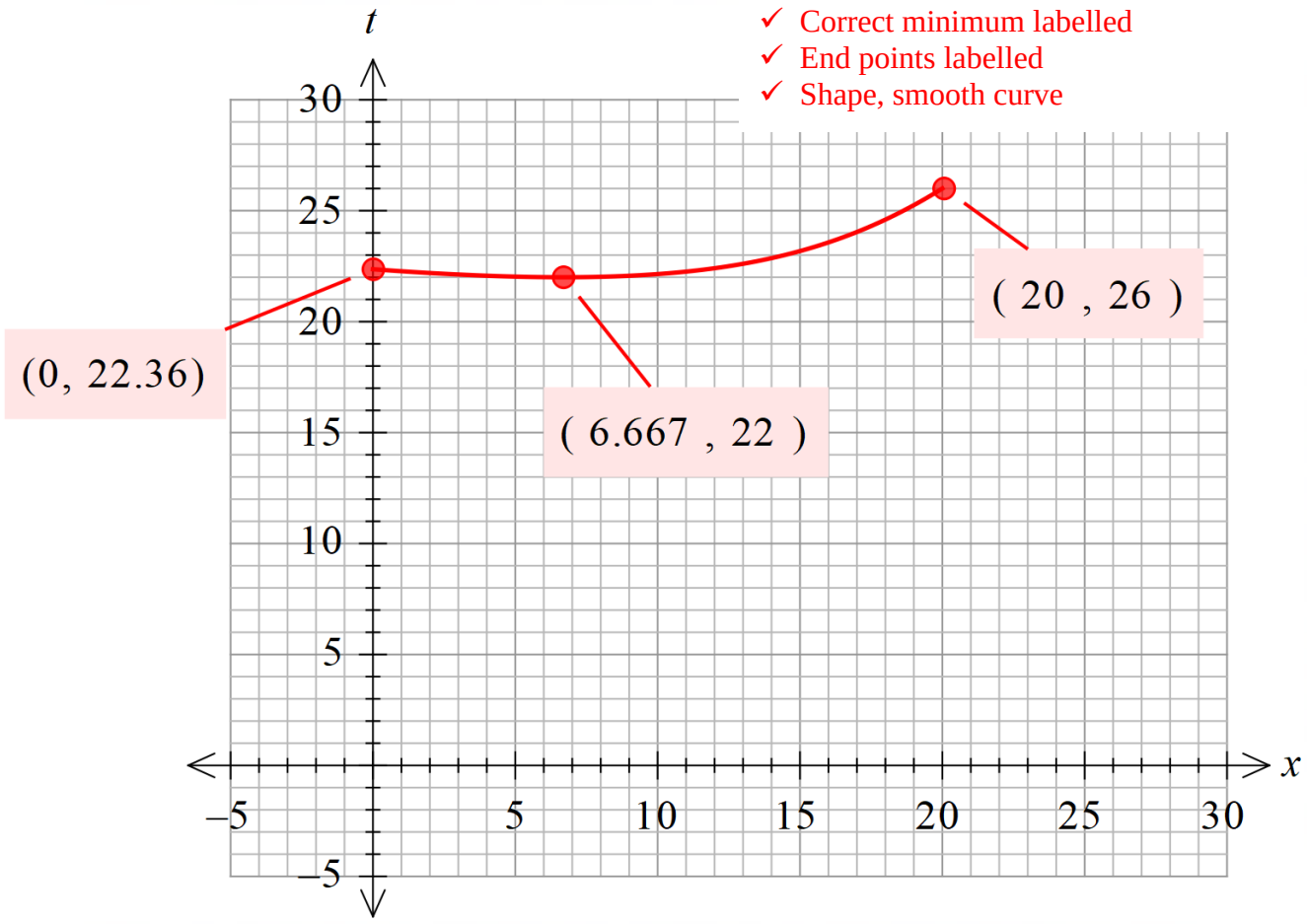
c) Solve the equation $\frac{dt}{dx} = 0$ for $0 \le x \le 20$

$x = \frac{20}{3}$

✓ Correct value

d) Graph $t(x)$ for $0 \le x \le 20$ on the axes below, labelling key features:

✓ Correct domain
✓ Correct minimum labelled
✓ End points labelled
✓ Shape, smooth curve



e) Hence, find the minimum time to complete the journey and state where the man should leave the riverbank to start swimming.

The minimum time will be 22 seconds and the man should leave the bank 6.67m from A

✓ States minimum time
✓ States distance from A to leave the